Control system for the model of an ankle prosthesis

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***Abstract*—The present work details the design of a control system for the model of an ankle prosthesis, capable of controlling the movement of the foot during each phase of the normal gait, controlling its angular position, for a low intensity walk on a flat surface. This system represents a stage for macro research projects in the development of robotic prostheses, specifically for ankle prostheses for cases of transtibial amputation.**

For the design of the control system, first the data were obtained of the trajectory, angular position, during the gait, and with the help of MatLab the analysis of the signals was carried out getting a mechanical model and a mathematical model. Using several methods control systems were designed that was applied to the mathematical model, to can compared the signal of response to a trajectory of reference and choose the best system that allow controlling the movement of an ankle prosthesis during a normal walk.

***Keywords*—Biomechanics; control; mathematical model; prosthesis; ankle-foot.**

I. Introduction

Some studies have indicated that one of main functions of the human ankle is provide the right energy for the forward progression of the body. Therefore, passive prostheses present solutions with limited expectations as far as personal aesthetics and functionality are concerned. People with a mechanical prosthesis exhibit non-symmetrical gait patterns and a high rate of metabolic energy expenditure [1].

Searching to improve the quality of life of people with amputations of the lower limb, has opted to resort to the use of biomechatronics prosthesis to solve the problem, that have been designed and built according to certain standards, these prostheses are on sale abroad at prices that are out of reach of the economic resources earmarked for public health in our nation. The use of these prostheses has been considered as an integral and definitive solution.

This project aims to simulate a control system for the movement of an ankle prosthesis during the gait, this will achieve greater functionality, comfort, naturalness, and energy saving for the user during the human gait, that is economically accessible and giving you a better quality of life.

II. Analysis of the Human Gait

The normal gait is a biped locomotion mode that is mainly performed by humans. During the gait, the periods of monopodal and bipodal support, this allows the displacement of the center of gravity of the human body with a lower energy expenditure to any other form of human locomotion [2], where the weight of the body is distributed alternately by both legs. The gait is composed of steps that make strides, that is also called the basic cycle of the gait, this is equivalent to two steps.

To analyze the human gait the cycle of gait is studied, a complete cycle is divided into two phases, the support phase that represents 60% of the cycle and the swing phase the remaining 40%, there is also the double-stand period that is when the two feet are in contact with the floor, it is presented at the beginning and at the end of the support phase (see Fig. 1) [3]. The cycle of the gait begins when the foot comes in contact with the ground, heel strike, and ends when the same foot comes back in contact with the ground [4], [5].

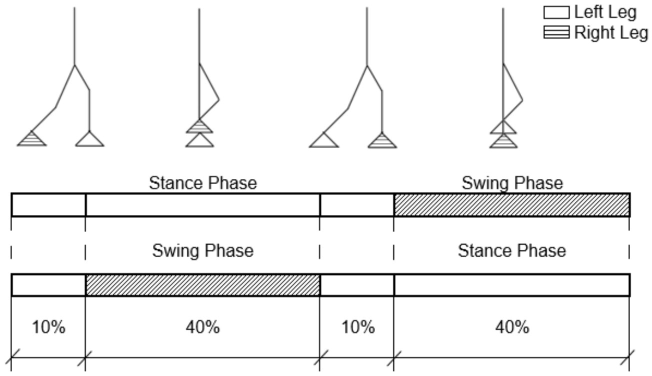

Fig. 1 Normal gait cycle.

A. Phases of the Human Gait

The support phase can be divided in three sub-phases: controlled plantarflexion (CP), controlled dorsiflexion (CD) and powered plantarflexion (PP); While the swing phase is divided in: initial, middle and final swing [5], the angles of action of these phases are described in Table I [6]. For this work, four phases are taken in account that are the sub-phases of the support phase and the swing phase (see Fig. 2) [7], that are described below:

1) Controlled Plantarflexion (CP):

CP begins with the heel strike with the ground surface and goes until the foot is fully in contact with the ground. Thus, the CP can be considered as a linear spring response where the torque is proportional to the position of the ankle [8].

2) Controlled Dorsiflexion (CD):

CD begins whit the foot-flat in contact with the ground until to where the angle that form the foot with the leg reaches its maximum state of dorsiflexion. can be described as a nonlinear spring where the force increases with the increase of the angular position of the ankle. During CD the ankle mainly stores elastic energy to propel the body upward [7].

3) Powered Plantarflexion (PP):

PP starts after CD until the toe-off of the ground. In this phase the elastic energy accumulated during CD is discharged, to reach the last posture before the swing phase [7].

4) Swing Phase (SP)

The swing phase is equivalent to 40% of the cycle of the gait and begins in the toe-off of the ground and ends in the heel strike of the same foot with the surface, in this phase the user's foot is raised to avoid the drag of the foot and the position of the foot is restored so that the first contact with the ground is with the heel (see Fig. 2) [7], [4].

TABLE I
ACTION ANGLES FOR NORMAL GAIT PHASES.

Normal Gait Phases		Action Angles*	
	Controlled Plantarflexion (CP)	0º	-15º
Support phase	Controlled Dorsiflexion (CD)	-15º	15º
	Powered Plantarflexion (PP)	15º	-25º
Swing Phase (SP)		-25º	0º

* Approximate average values

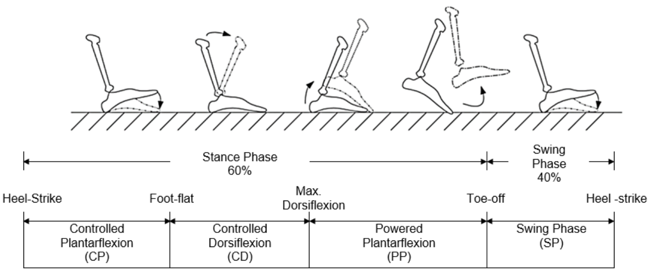

Fig. 2 Phases during a normal gait cycle.

III. MECHANICAL MODEL

For the development of the model used the tools of Simmechanics, in Simulink, thus the system leg-foot-ankle, can be analyzed in a dynamic way, taking as rigid links articulated with a constant length to the lower extremity.

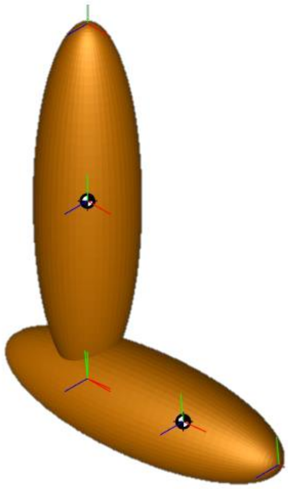

Fig. 3 Model in Simmechanics of the leg-foot-ankle system.

In the block diagram in Simmechanics, that can be seen in Fig. 3, the leg is represented so that it is fixed to the knee joint, for it is used the Weld tool, that fixes a link end of the leg, this way this end will have no degree of freedom.

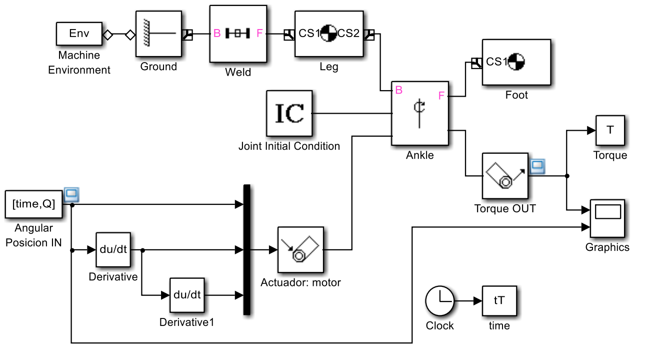

Fig. 4 System model block diagram in Simmechanics.

For the configuration of each link, the data to be entered are the mass, the location of its center of gravity (CG), the moment of inertia that must remain constant during the movement, and it is necessary to enter the coordinates systems (CSs), these coordinates indicate the location of the extreme points of the links. The anthropomorphic data of the body segments such as the length, weight and moments of inertia for each link can be seen in the Table II [5].

TABLE II
ANTHROPOMETRIC DATA OF THE BODY SEGMENTS OF THE LOWER EXTREMITY.

		Leg	Foot
Length		$(0.246)H$	$(0.152)H$
Model Length *		$0.39852\ m$	$0.24624\ m$
Mass		$(0.0465)M$	$(0.0145)M$
Model Mass **		$3.1155\ Kg$	$0.9715\ Kg$
Moment of inertia	x	$0.0369\ Kg.m^2$	$0.0037\ Kg.m^2$
	y	$0.0369\ Kg.m^2$	$0.0037\ Kg.m^2$
	z	$0.00268\ Kg.m^2$	$0.0008\ Kg.m^2$

* H=1.62 m, ** M=67 Kg.

The output will be given by the joint sensor block that allows to observe several outputs, such as the torque during the movement of the ankle. The scope block (Graphs) shows the graphs of the angular position and the torque of the ankle model on the Fig. 5.

IV. Mathematical Model

A mathematical model that was used in this research was made by Dr. Gill in 1998, see [9]. In this case the results obtained with this model will be compared with those obtained with the previous model. This mathematical model was adapted from the model of three degrees of freedom in the sagittal plane that can be observed in Fig. 6 [9]. It describes the movement of the lower extremity in the swing phase of the gait that was obtained by applying the equations of Euler Lagrange.

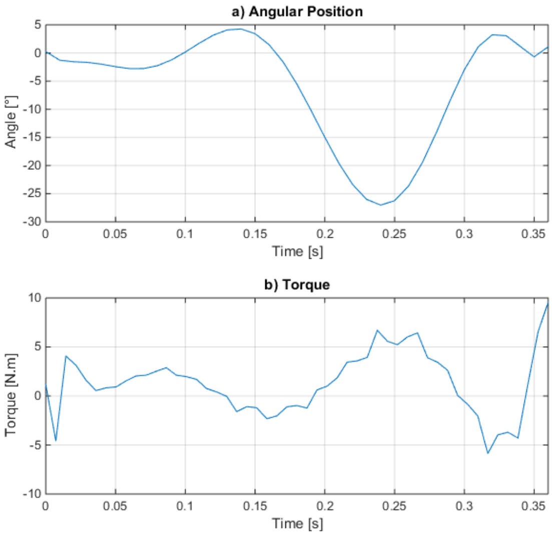

Fig. 5 (a) Angular path of gait, (b) Ankle torque of the model in Simmechanics.

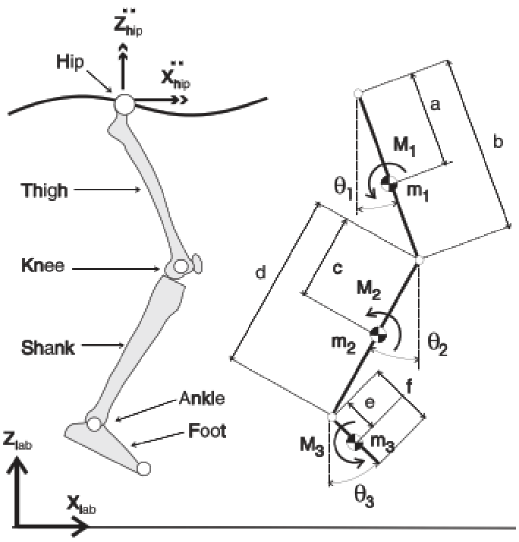

Fig. 6 Model on the sagittal plane of the leg swing.

In this case the system to control considers the leg and foot segment as you can see in Fig. 7 [5].

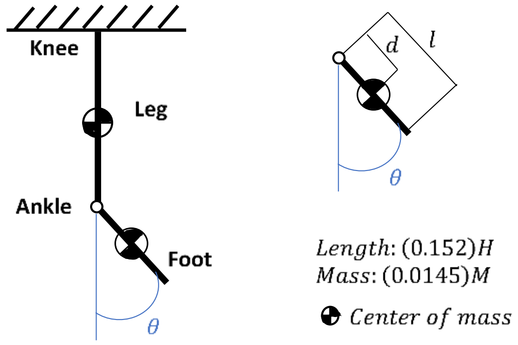

Fig. 7 Diagram of the free body of the kinetic foot model in the sagittal plane.

In Simmechanics each segment of the leg and foot are represented as rigid links and taking the ankle as a joint, Gill's study equation is simplified to the following expression:

$$(J_c + md^2)\ddot{\theta} + k\dot{\theta} + mgdSen\theta = T_d \qquad (1)$$

Where:

θ generalized coordinates

J_c inertia of the body

m m ass of the whole system

d distance from the joint to the center of mass of the foot

T_d motor torque

The torque generated by the ankle varies from the position (Newton's second law for rotational movement). It should be considered a total torque of T_t handling considering the torque of the prosthesis due to the friction of the joint T_c, quedando definido T_t as:

$$T_t = T_d - T_c \qquad (2)$$

In Simulink is represented the equation, with the same data of the model of Simmechanics. The torque $T_c = 1.46\ Nm$ and $k = 0.5$ were defined in the mathematical model during the simulations in Simulink to adapt the output of angular trajectory to that obtained in Simmechanics (see Fig. 8).

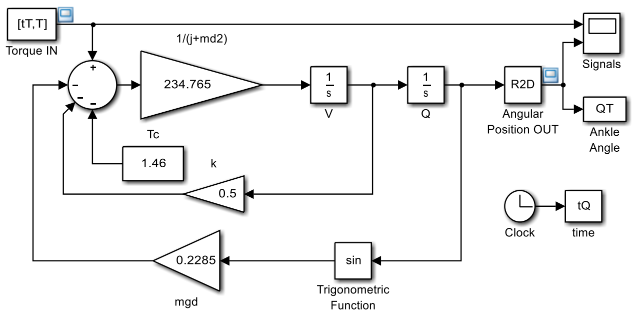

Fig. 8 Mathematical model equation in Simulink.

The equation represented in Simulink has as input data the torque, obtained depending on the time of the simulation in Simmechanics, from this data is obtained the angular position of the ankle (see Fig. 9).

When comparing the entry path in Simmechanics with the output trajectory of the mathematical model in Simulink, to be observed in Fig. 10. It is noted that they tend to a natural gait cycle, considering that, in a normal walk, the range of movement of the ankle is approximately 15° of maximum dorsiflexion and 30° of plantar flexion. However, it can be observed that there is a greater difference between the angle of entry and the angle of exit during the support phase that in the swing phase.

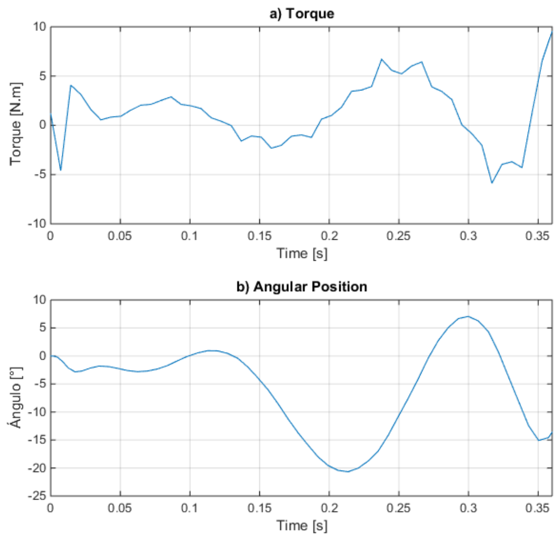

Fig. 9 (a) Model ankle torque in Simmechanics, (b) Angular trajectory of the mathematical model in Simulink.

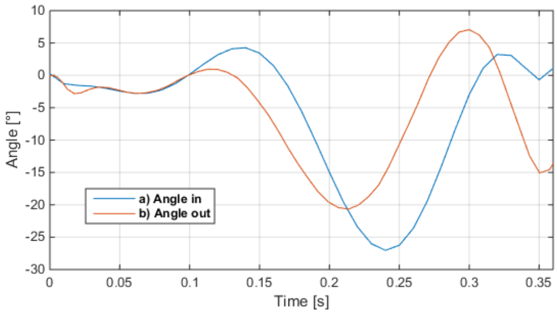

Fig. 10 (a) Angular trajectory of the ankle of entry to the model in Simmechanics. (b) Angular trajectory of the exiting ankle of the mathematical model in Simulink.

V. Plant and Engine Model

The representation of the linear system in the state space of the model plant and engine is given by the equation of states reduced by Villa Parra, 2011 shown below [5].

$$\begin{bmatrix}\hat{\dot{x}}_1\\ \hat{\dot{x}}_2\\ \hat{\dot{x}}_3\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{-mgd}{J_c+md^2} & \dfrac{-k}{J_c+md^2} & \dfrac{K_t}{J_c+md^2} \\ 0 & -\dfrac{K_e}{L} & -\dfrac{R}{L}\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \hat{x}_3\end{bmatrix} + \begin{bmatrix}0\\0\\ {}^{1}/{}_{L}\end{bmatrix}u(t) \tag{3}$$

$$\hat{y} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \hat{x}_3\end{bmatrix}$$

With the help of MatLab, the values of the parameters of Tabla III are replaced and the ss2tf, command is used, which converts a representation into the state space of a system in an equivalent transfer function (4).

TABLE III
SYSTEM PARAMETERS

Plant parameters	Engine parameters *	
$J_c = 0.0037\ Kg.m^2$	Armature constant	$Kt = 0.113\ Nm/A$
$m = 0.9715\ Kg$	Cte. of electromotive force	$Ke = 0.0115\ V/rpm$
$d = 0.246\ m$	Armor Resistance	Ra = 13 ohms
$k = 0.5$	Armature inductance	$La = 0.01\ henrios$
$g = 9.8\ m/s$		

* Nominal values of a rotating angular motor RA29 BEI Kimco

$$H = \frac{180.8}{s^3 + 1308\ s^2 + 10441\ s + 48722} \tag{4}$$

VI. Control Systems

The simulation of a gait cycle is performed with the controllers designed and through the comparison of the signals obtained from the systems, the control system that most adapts to the desired control objective is chosen.

A. PID Control System Design

There will be two types of control systems, starting with the PID type, will be used the method of tuning of Ziegler-Nichols, and the method of the place geometric of the root.

1) First method of tuning Ziegler-Nichols

This method can be applied because the response of the plant to a unitary step input shows a curve in S shape (see Fig. 11).

Drawing a tangent line at the inflection point of the curve, the intersections of this tangent are determined with the axis of time and the horizontal line marking at which point the curve stabilizes. For which we obtained the values of $L = 0.0567$ and $T = 0.3416$.

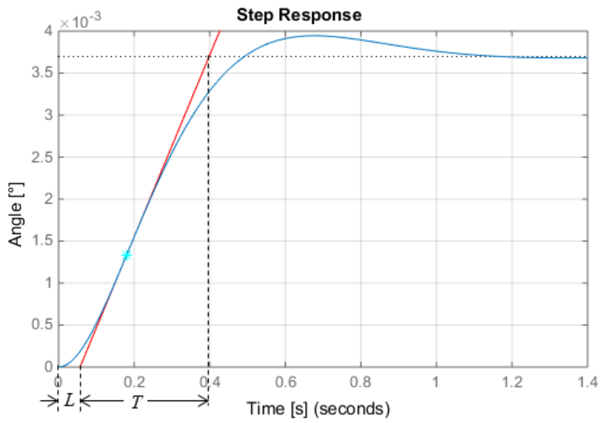

Fig. 11 Step response, tangent line to the tipping point.

By applying the Ziegler-Nichols tuning rules based on the step response of the plant (first method), you get the values for K_p, T_i y T_d (5), (6), (7), thus the parameters for the PID controller (9).

$$K_p = 1.2\frac{T}{L} = 7.2296 \tag{5}$$

$$T_i = 2L = 0.1134 \tag{6}$$

$$T_d = 0.5L = 0.0284 \tag{7}$$

$$Gc(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{8}$$

$$K_d = 0.21 \quad K_p = 7.23 \quad K_i = 63.75 \tag{9}$$

Submitting the plant with the PID controller to a step signal, it shows a response signal (see Fig. 12), where the setting time is $t_s = 16.2s$.

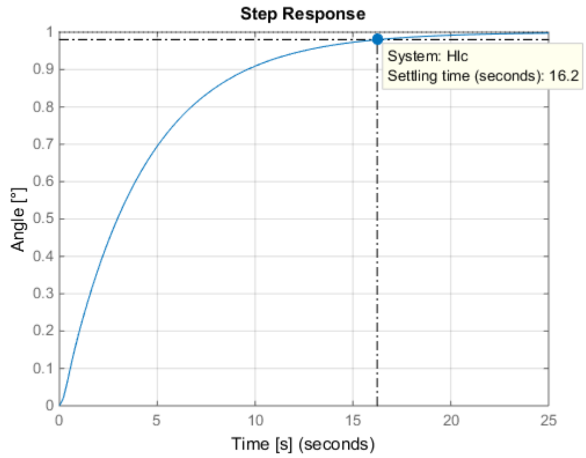

Fig. 12 Response of the plant with PID controller to a step input.

In Fig. 13 shows the implementation of the PID control in Simulink block diagram, using the PID Controller block in which the calculated parameters are placed.

The signals obtained after implementing the PID control system in the plant in Simulink can be seen in Fig. 14. Where the response obtained about the angular trajectory of the ankle is very slow, so it is necessary to design a control system by other methods.

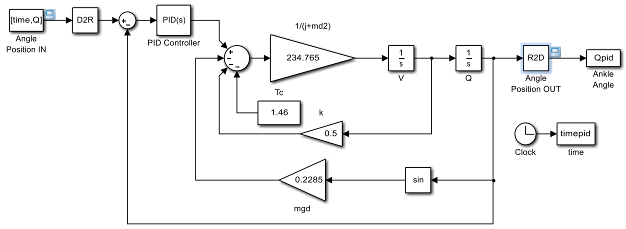

Fig. 13 Block diagram of the PID control system.

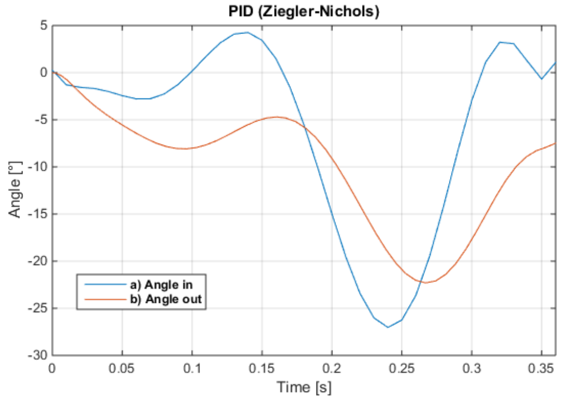

Fig. 14 (a) Reference of the angular trajectory of the ankle. (b) Angular trajectory of the ankle with PID control (Ziegler-Nichols method).

2) PID method of Place Geometric of the Root

The design of this PID controller was performed out considering the following parameters as a starting point: a damping coefficient, $\mu = 0.94$, and a setting time, $t_s = 0.2s$ because you want to get a quick response to the input signal of the System, whereupon a system oscillation frequency was obtained (ω_n), using the equation of the time of establishment of second order systems.

$$t_s = \frac{4}{\mu\omega_n} \tag{10}$$

$$\omega_n = \frac{4}{\mu t_s} = \frac{4}{(0.94)(0.2)}$$

$$\omega_n = 21.28\frac{rad}{s}$$

The location of the desired poles (P_d) in closed loop can be determined starting from (11).

$$P_d = -\mu\omega_n \pm j\omega_n\sqrt{1-\mu^2}$$
$$P_d = -20 \pm j\,7.26 \tag{11}$$

Using a shape controller (12).

$$G_c = \frac{(s+a)(s+b)}{s} \tag{12}$$

Where the first zero is assumed, $a = 1$ to proceed to calculate the second zero with the condition of magnitude and angle of the LGR; With the command angle in MatLab you get the angle Φ in radians, without b of the form (13).

$$\Phi = angle\left(H * \frac{(s+a)}{s}\bigg|_{s=P_d}\right) \tag{13}$$

$$\Phi = 44.64°$$

Therefore, there is a deficiency of 135.36°, which will be compensated by the remaining zero which is calculated in the following way (14); and finally replacing the variables and matching the regulator with the characteristic equation of a PID controller you get the parameters K_d, K_p y K_i (15).

$$\measuredangle(s+b)|_{P_d} = 135.36° \tag{14}$$

$$\measuredangle(-20 \pm j\,7.26 + b) = 135.36°$$

$$\tan^{-1}\left(\frac{7.26}{b-20}\right) = 135.36°$$

$$b = \frac{7.26}{\tan(135.36°)} + 20$$

$$b = 12.65$$

$$\frac{K_d s^2 + K_p s + K_i}{s} = \frac{(s+a)(s+b)}{s} \tag{15}$$

$$K_d = 1 \quad K_p = 13.65 \quad K_i = 12.65$$

Applying the plant with the controller to a step signal was obtained the answer that is observed in the Fig. 15, where you can see that the response time is very high, $t_s = 85.8s$.

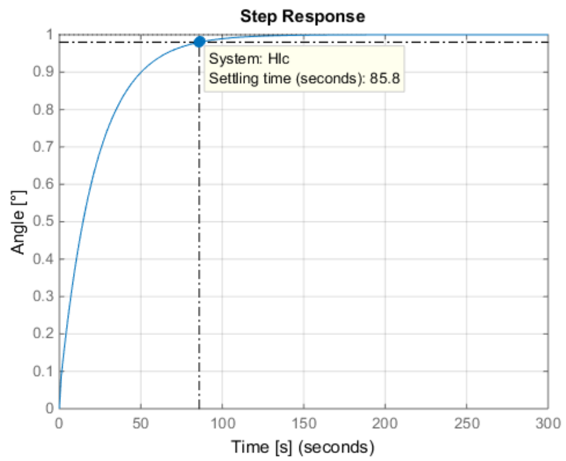

Fig. 15 Response of the plant with PID controller to a step input.

Placing the parameter values in the block diagram with the PID controller (see Fig. 13). You get an output signal that you see in Fig. 16, along with the reference signal.

Because there is an error between the reference signal and the output signal, as there is a delay in response it is decided to reduce the setting time (t_s) of the signal, manually modifying the frequency of oscillation of the system (ω_n), because it is inversely proportional to the t_s (10).

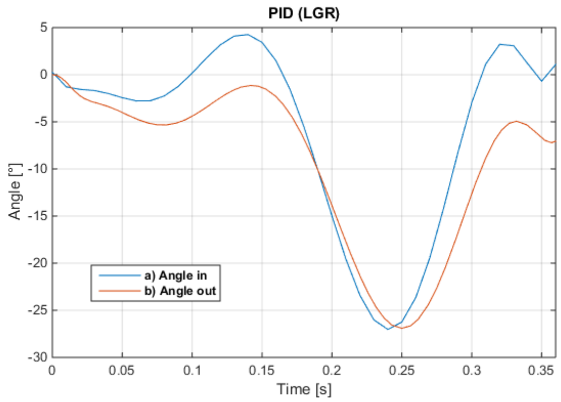

Fig. 16 (a) Reference of the angular trajectory of the ankle. (b) Angular trajectory of the ankle with PID control (LGR method).

This value is increased by observing the behavior suede a step signal reaching $\omega_n = 663$, a value that offers a shorter time of establishment without having a maximum overdrive over 5% and thus being able to maintain a stable signal, maintaining the part Real of the desired pole (11), then you have a new pole, which is:

$$P_d = -20 \pm j\,226.2$$

Performing the same procedure before seen, it has as a regulator

$$G_c = \frac{(s+a)(s+b)}{s} = \frac{(s+1)(s+701.25)}{s}$$

Equalizing the regulator with the characteristic equation of a PID controller (15), the following values were obtained:

$$K_d = 1 \quad K_p = 702.25 \quad K_i = 701.03$$

Applying a step input signal (see Fig. 17) You get a setting time of $t_s = 3.39s$, much lower than the previously obtained, and you can see that there is an overdrive of $M_p = 4.99\%$ in $t = 0.30s$ allowing the system to grant a faster response without destabilizing the system.

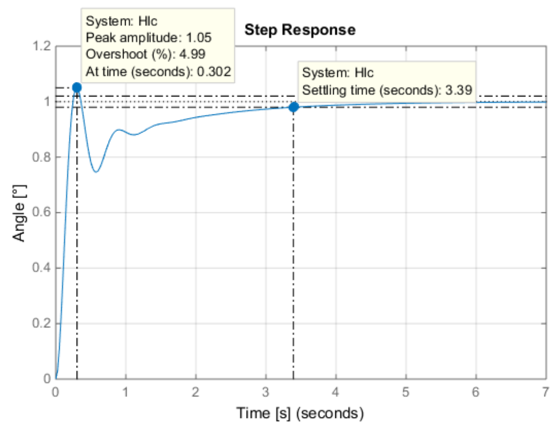

Fig. 17 Step response of the plant with PID controller (modified ω_n).

Presenting a response signal with a minimum error to the reference signal compared to the one obtained previously without modifying the oscillation frequency of the system (see Fig. 18).

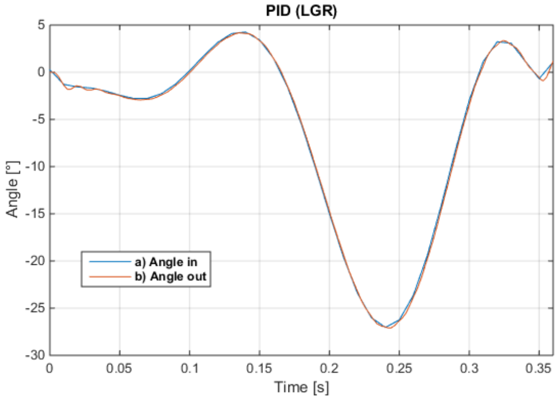

Fig. 18 (a) Reference of the angular trajectory of the ankle. (b) Angular trajectory of the ankle with PID control (LGR method).

B. State Feedback Control System Design

The design of state feedback control systems was performed observing the responses of each system to obtain the best output signal, beginning with the design of state feedback, followed is added an observer, and finally is design the Integral action.

1) Feedback of states

The system is designed in accordance with the State Feedback control scheme [10] (see Fig. 19), where K is the status feedback gain matrix.

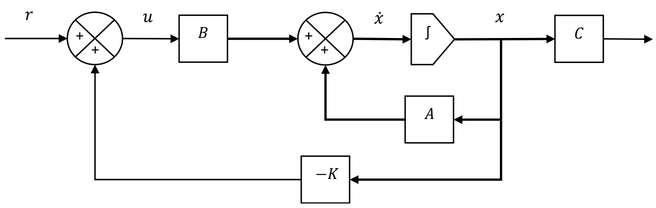

Fig. 19 Scheme of a state feedback control system.

Using the pole mapping method, the required starting parameters are proposed, a buffer coefficient of $\mu = 0.94$ and a time of establishment $t_s = 0.2$ so the obtained poles are $P_d = -20 \pm j7.26$, the third pole must be in a position of such so that the first poles act as dominant, therefore, the third pole is placed in such a way that it is remote, in $P_3 = -2000$.

To find the values of the matrix K is made use of MatLab with the command place (A, B, P), where A and B are matrices of the transfer function and P are the desired poles.

Obtained the matrix $K = [4585.8 \quad 354.79 \quad 7.32]$, is implemented in the diagram of blocks of the plant in Simulink (see Fig. 20).

Applying the control system designed to a step input, you get the signal that can be seen in Fig. 21, which is very small, with maximum peak of $M_p = 0.0151\%$ and a $t_s = 0.242\ s$.

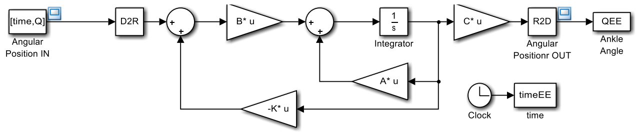

Fig. 20 Block diagram with State feedback.

Applying the control system designed to a step input, you get the signal that can be seen in Fig. 21, which is very small, with maximum peak of $M_p = 0.0151\%$ and a $t_s = 0.242\ s$.

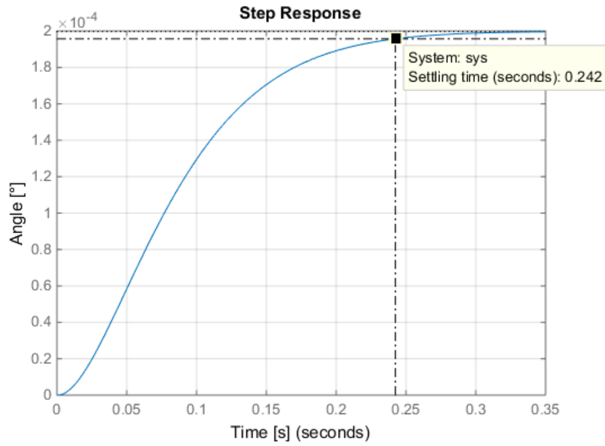

Fig. 21 Response of the plant with state feedback controller to a step input.

In the output signal of the system (see Fig. 22), it is observed that it has a trajectory similar to the desired one, but with very small values, that compared with the reference signal it seems that it has a value of 0 ° (see Fig. 23).

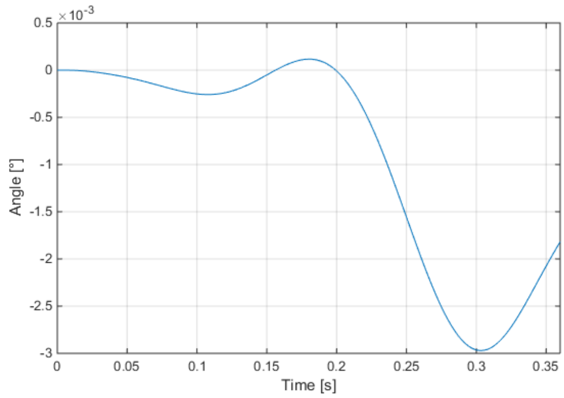

Fig. 22 Response to the reference signal applied to the plant with state feedback controller

After obtaining this answer that is not suitable for the control system because as it is observed you do not obtain a response signal equal to the reference signal is proceeded to the design of a observed.

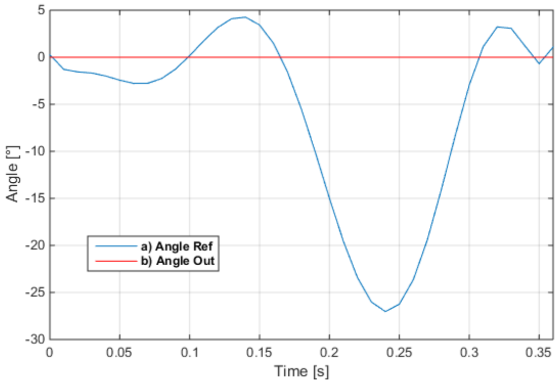

Fig. 23 (a) Reference of the angular trajectory of the ankle. (b) Angular trajectory of the ankle with state feedback control (see Fig. 22).

2) Feedback of States with Observer

"The estimation of non-measurable state variables is commonly referred to as observation. A device (or a computer program) that estimates or observes state variables is called a state observer, or simply an observer." [10].

This design is applied assuming the case where the system state variables are not measurable, using the gain matrix $K = [4585.8 \quad 354.79 \quad 7.32]$ previously calculated, therefore, the observer is applied in the form indicated in the diagram of blocks (see Fig. 24) [10], to try to obtain a better signal than that obtained only with the feedback of states.

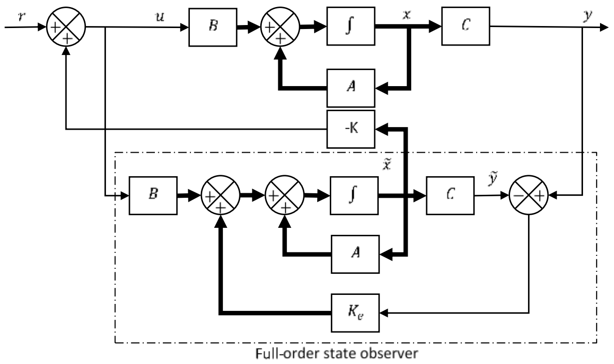

Fig. 24 Block diagram of the state feedback control system with observer.

Where K_e is the matrix of gain of the observer, to obtain this gain is made use of MatLab with the command Ke=place (A', C', P); where P are the poles of the observer which is desired, for this case it is chosen enough high mind to make them faster than the dominant pole, $P = [-3000 \quad -3100 \quad -3200]$, thus obtaining the matrix of the gain.

$$K_e = [7.99e03 \quad 1.84e07 \quad 3.22e09]$$

Implementing the observer in the control system (see Fig. 25) and subjecting to a step input signal a signal similar to the previous design is obtained which did not have an observer (see Fig. 26), giving us the same maximum peak of $M_p = 0.0151\%$.

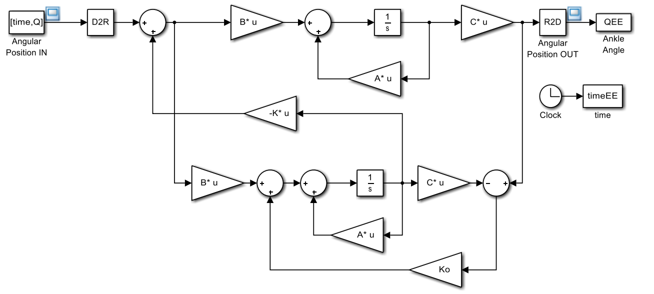

Fig. 25 Block diagram with the implementation of the observer in Simulink.

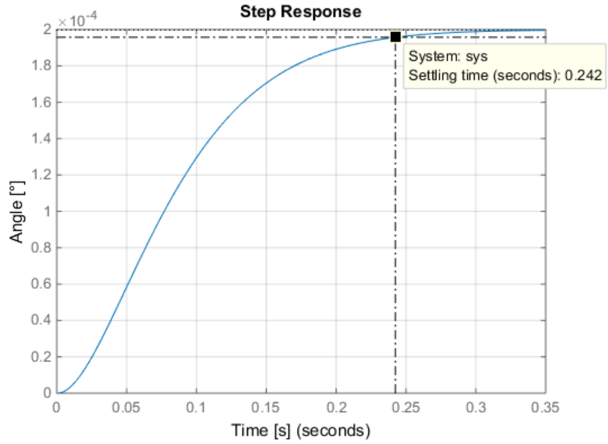

Fig. 26 Response to a step signal of the plant with observer.

Likewise, the signal obtained is maintained in a similar way to when the observer was not used, having a large difference in scale between the reference signal and the signal with the controller as shown in Fig. 27, in this way not obtaining good results, we proceed to make the feedback of states with integral action.

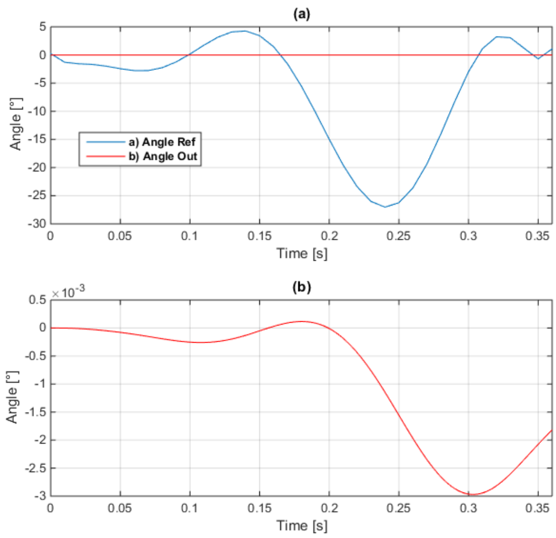

Fig. 27 Response to the reference angular trajectory. (a) Comparison between the reference signal and the output signal, (b) output signal observed on a larger scale.

3) State Feedback and Integral Action

A constant reference tracking scheme is introduced with rejection properties of constant input disturbances by increasing the number of state variables in the plant, thus adding a new state x_a that integrates the error of tracking (see Fig. 28).

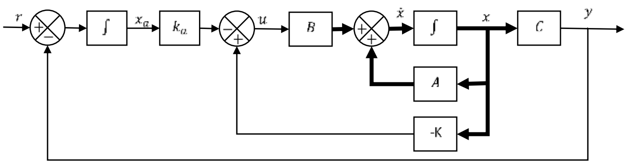

Fig. 28 Block diagram of the state feedback control and integral action system.

To have the model with the integrator, is increased to the equations of state an extra state that is the output of the integrator.

$$\begin{bmatrix} \dot{x}_a \\ \dot{\widehat{x_1}} \\ \dot{\widehat{x_2}} \\ \dot{\widehat{x_3}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \dfrac{-mgd}{J_c + md^2} & \dfrac{-k}{J_c + md^2} & \dfrac{K_t}{J_c + md^2} \\ 0 & 0 & -\dfrac{K_e}{L} & -\dfrac{R}{L} \end{bmatrix} \begin{bmatrix} x_a \\ \widehat{x_1} \\ \widehat{x_2} \\ \widehat{x_3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L \end{bmatrix} u(t) \tag{16}$$

$$\hat{y} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_a \\ \widehat{x_1} \\ \widehat{x_2} \\ \widehat{x_3} \end{bmatrix}$$

In the same way replacing the parameters of the matrices and using the command place in MatLab, you get the profit matrix K. For which it is necessary to locate a pole to loop closed for each pole in the plant, the poles are used previously calculated and it is located a pole in-2500 being this faster than the rest of the poles.

With the gains obtained for $k_a = [1.2517e07]$ and $K = [1.12e06 \quad 28448 \quad 32.33]$, it is implemented in the block diagram in Simulink (see Fig. 29).

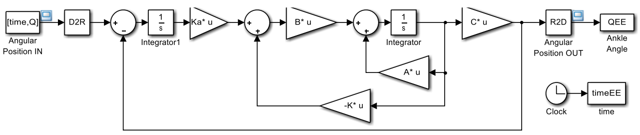

Fig. 29 Block diagram of the state feedback control system and integral action.

When applying the control system to a unit step input signal (see Fig. 30), it can be observed that it has a time of establishment of $t_s = 0.2s$ and when applying the reference signal of the angular position to the system (see Fig. 31), a signal was obtained that tends to follow The reference trajectory, but with a delay in the response, because of this some modifications are made to obtain a better answer.

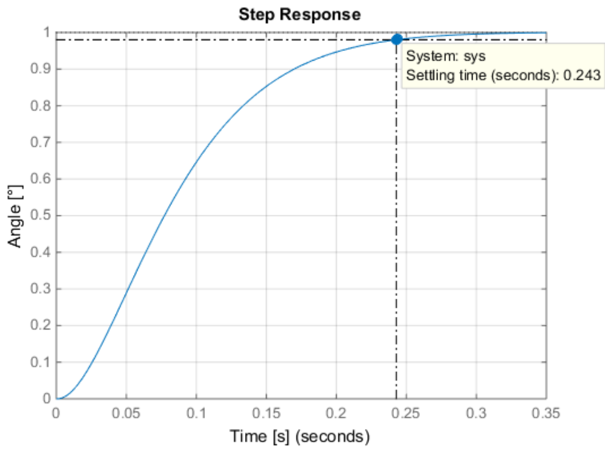

Fig. 30 Response of the state feedback control system with integral action to a step input.

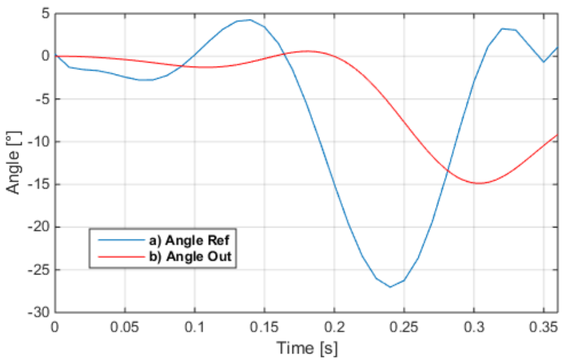

Fig. 31 (a) Reference of the angular trajectory of the ankle. (b) Angular trajectory of the ankle with state feedback control and integral action.

It modifies manually the imaginary part of the desired pole, to have a better answer, because in this way the signal response time is reduced (t_r).

$$t_r = \frac{\pi\theta}{\omega_d} = \frac{\pi\theta}{\omega_n\sqrt{1-\mu^2}} \tag{17}$$

Because the natural frequency buffered (ω_d), can be identified as the imaginary part of the equation characteristic to find the desired pole, modifies only this imaginary part of the desired pole to obtain a value that provides a faster response and stable, choosing a value which shows the desired response signal.

$$P_d = \left(-\mu\omega_n \pm \omega_n\sqrt{1-\mu^2}\right) \tag{18}$$

$$P_d = (-\mu\omega_n \pm \omega_d)$$

$$P_d = (-20 \pm 80)$$

Performing the previous procedure, with the modification of the poles.

$$P_1 = (-20 + 80)$$

$$P_2 = (-20 - 80)$$

$$P_3 = (-2000)$$

$$P_4 = (-2500)$$

The gains obtained with the modifications made are $k_a = [1.88e08]$ and $K = [1.27e06 \quad 28483 \quad 32.32]$, and them in the system (see Fig. 29), it can be observed that by reducing the response time and applying a step signal, the response signal is sub-dampened with an overdrive of $M_P = 45.4\%$, that manages to stabilize in a $t_s = 0.17$s (see Fig. 32).

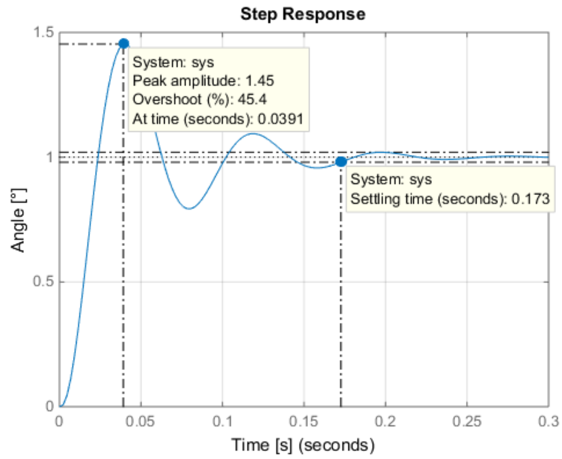

Fig. 32 Response to a single step signal from the state feedback controller and integral action.

The response obtained to the reference signal is observed that it tends to follow its trajectory without having a greater error, presenting a quicker response than the previous one (see Fig. 33). So, the modification to the desired pole was of great help to this design.

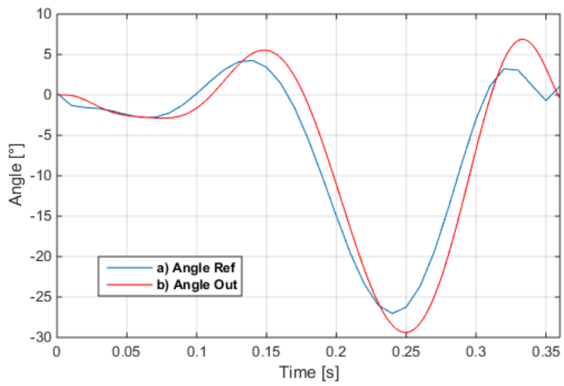

Fig. 33 (a) Reference of the angular trajectory of the ankle. (b) Angular trajectory of the ankle with state feedback control and integral action (modified pole).

VII. Results

In Fig. 34 is summarized, the reference signal of the angular trajectory of the ankle along with the different signals of outputs obtained with each of the designs of controllers made.

The best control systems offering an angular trajectory of the appropriate ankle are the PID control system by the place geometric of the root method and the state feedback control system and integral action. Because the signals obtained from the other designed systems have a much greater error, because they do not follow the desired trajectory and there is a high difference between the desired input values and the output values.

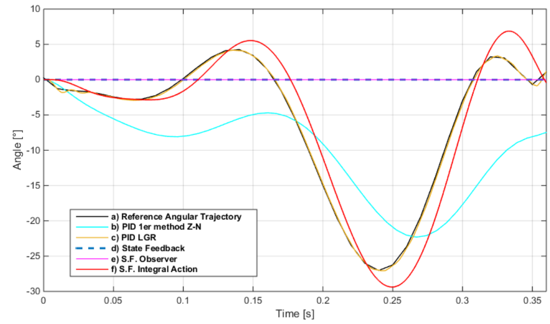

Fig. 34 Reference signal of the angular trajectory of the ankle-foot and signals obtained from the control systems designed.

The PID control system by the place geometry of the root method shows the best output signal, because it was modified by increasing the attenuated natural frequency (ω_n) n order to reduce the setting time (t_s) of the signal, always keeping the actual part of the desired pole. resulting in a response of the angular trajectory of the ankle more faithful to the reference signal.

On the other hand, the system of control by feedback of States and Integral action shows a signal that is within the range of a cycle of a normal human gait, Fig. 32 is the system response to a unitary step input, where you can see that it is a sub-buffered response with a maximum impulse of 45.4%, which produces the output signal shown in Fig. 33.

In Fig. 35 it is possible to observe the signals obtained from the two systems that offer a better response by submitting them to a step signal, with which one can notice a difference, in relation to the maximum on impulse and the time of establishment of each one; due to these signals can be said that the system of control by feedback of states with integral action offers us a time of establishment much greater but with a great overdrive which distorts the signal of output, on the other hand the system of control PID by the place geometric of the root has a small envelope impulse which allows to obtain a more stable signal of response and with a low setting time is considered that this system is the most efficient, these data can be compared in a better way in the Table IV.

TABLE IV
COMPARATIVE TABLE OF IMPLEMENTED DRIVERS

Control System	Setting time t_s (s)	Maximum over-impulse M_p (%)
PID 1er method Z-N	16.2	0
PID LGR	3.39	4.99
Feedback from states	0.24	0
F. S. Observer	0.24	0
F. S. Integral Action	0.17	45.4

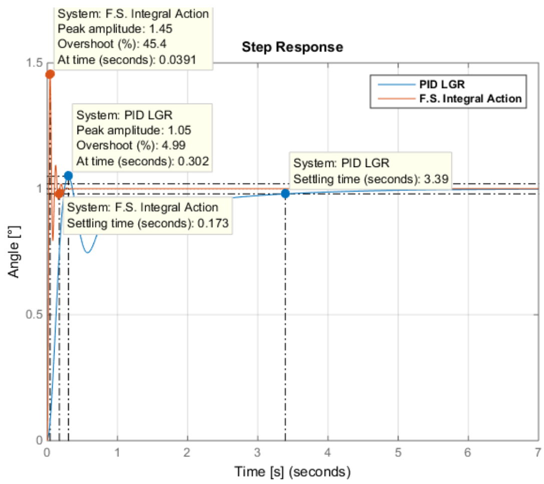

Fig. 35 Step signal of PID control systems by the place geometric of the root and feedback of states with integral action.

VIII. Conclusion

The study of the dynamics of the movement of the ankle during a low-intensity gait unveils the different phases that exist during the cycle, allowing to identify the angular trajectory of the data obtained from a normal walk of low intensity, the which was used as a reference signal for the design of the different control systems carried out.

Working with various methods for the design of control systems and making modifications in some parameters made it possible to determine before which controller, the system provides a better response to a reference signal by obtaining a response fast, with minimal error in stable state and low times of establishments.

The simulation of the systems of control to carry out them with the help of MatLab allowed, both to obtain the simulation of the mechanical and mathematical model and to compare the different answers achieved with the 5 different strategies of control making possible to realize the respective modifications and observe the results in a more efficient way.

When comparing the obtained results, it was observed that the PID control system designed using the place geometric of the root was the most efficient among all, with a maximum overdrive of $M_p = 4.99\%$ and a time of establishment of $t_s = 3.39s$, showing that it can become very effective as to your response to a signal.

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